

# CCD Photometric Observations of the Galactic Globular Clusters NGC 1904 and NGC 6341

Alok C. Gupta

Harish-Chandra Research Institute, Chhatnag Road, Jhansi, Allahabad – 211 019,  
India

**email:** agupta@mri.ernet.in

## ABSTRACT

We present multi-colour broad band CCD photometric observations of a few thousands of stars located region of the galactic globular clusters NGC 1904 and NGC 6341. These observations are used to generate their deep colour magnitude diagrams (CMDs) to study overall morphologies of sub giant, red giant, asymptotic giant and horizontal branches. We also determine the main sequence morphology of the NGC 6341. A comparison of their CMDs with the theoretical isochrones given by Demarque et al. (1996) indicates an age of 14 and 16 Gyr for NGC 1904 and NGC 6341 GGCs respectively.

We present a new method for artificial add star experiment. Using this method on the photometric data of galactic globular cluster NGC 6341, we determined the completeness factor in B, V and I pass bands. We have determined the luminosity function of the main sequence stars down to  $V \sim 21.0$  ( $M_V \sim 6.5$ ) from the V Vs (B – V) colour magnitude diagram of this cluster.

Using accurate parameters of 64 GGCs, we derived relations between galactocentric distance R, distance from the galactic plane  $Z_g$  and metallicity [Fe/H] of GGCs.

*PACS:*

**Keywords:** Galactic Globular Clusters: NGC 1904 (M 79), NGC 6341 (M 92); Photometry, Colour Magnitude Diagrams, Add Star Experiment, Completeness Factor, Luminosity Function

# 1 INTRODUCTION

Precise multi-colour photometric observations of the Galactic Globular Clusters (GGCs) provide valuable information about the late stages of stellar evolution in low-mass stars, as they are amongst the oldest objects in our galaxy. They are also useful for the study of galactic evolution. We have selected two GGCs namely NGC 1904 (M 79), and NGC 6341 (M 92) for the present study and provide basic informations about them in Table 1. A brief description of their earlier photometric studies is given below.

**Table 1.** Basic cluster parameters of the four galactic globular clusters included in the present study. The GGCs parameters are taken from Djorgovski (1993).

| Parameter                   | Cluster   |   |
|-----------------------------|---|---|
|                             | NGC 1904  | NGC 6341  |
| R. A. (2000.0)              | 05 <sup>h</sup> 24 <sup>m</sup> 10.6 <sup>s</sup> | 17 <sup>h</sup> 17 <sup>m</sup> 07.3 <sup>s</sup> |
| Dec. (2000.0)               | −24°31′27″  | +43°08′11″  |
| Gal. longitude (deg)        | 227.229   | 68.339  |
| Gal. latitude (deg)         | −29.351   | 34.859  |
| E(B−V) (mag)                | 0.01  | 0.02  |
| Distance (kpc)              | 13.0  | 7.5   |
| Concentration c             | 1.72  | 1.81  |
| log (R) (kpc)               | 1.28  | 0.96  |
| log (Z <sub>g</sub> ) (kpc) | 0.80  | 0.63  |
| [Fe/H]                      | −1.69   | −2.24   |

## 1.1 NGC 1904

First photoelectric photometric study of the cluster NGC 1904 is done by Goranskij (1976) and in the same year Alcaino (1976) published photographic BV colour-magnitude diagram (CMD) of 95 stars. The main features of the CMD are: the presence of a large sample of blue HB stars in contrast to lack of red stars. The

values of reddening  $E(B-V)$  and true distance modulus  $(m-M)_0$  are determined as 0.09 mag and 15.28 mag respectively. Stetson & Harris (1977) published UBV photographic data of 152 stars in the magnitude range  $13.0 < V < 17.2$ . They found very small value of foreground reddening  $E(B-V) = 0.01 \pm 0.01$  mag and reported the location of the HB at  $V_{HB} = 16.2 \pm 0.1$  mag. The cluster is therefore located at a geocentric distance of 13 kpc and at a galactocentric distance of 21 kpc.

First main-sequence (MS) photometry of NGC 1904 is presented by Harris et al. (1983) using SIT Vidicon Camera. They produced a BV CMD for 264 stars in the magnitude range  $16 < V < 22$ . They found the MS turn off at  $V \sim 19.4$  mag and using theoretical isochrones, derived age of the cluster between 12 to 18 Gyr. Cordoni & Auriere (1983) presented BV magnitudes for 161 stars in a  $1' \times 1'$  field centered on the cluster.

First CCD photometry of this cluster is carried out by Heasley et al. (1986). Their BV CCD photometric study upto  $V \sim 21.5$  mag is well below the MS turn off (MSTO). They found that the cluster is located at 20 kpc away from the galactic center. They derived cluster age as 16 Gyr. Gratton & Ortolani (1986) carried out BV CCD photometric study of 227 stars upto  $V \sim 21.5$  mag and derived the cluster age as 15 to 18 Gyr.

Ferraro et al. (1992) presented BV photographic photometry of 3188 stars down to  $V \sim 21$  mag. They studied the overall morphology of different branches of CMD of the cluster. They located RGB bump at  $V = 16.00 \pm 0.05$  mag and determined the mean metallicity,  $[Fe/H] = -1.60 \pm 0.20$ ; Horizontal branch (HB) at  $V_{HB} = 16.15 \pm 0.10$  mag; MSTO at  $V_{TO} = 19.60 \pm 0.20$  mag;  $\Delta V_{TO-HB} = 3.45 \pm 0.22$  mag;  $M_V(HB) = 0.70 \pm 0.07$  mag,  $(m-M)_V = 15.45 \pm 0.02$  mag which put the cluster at a distance of  $12 \pm 2$  kpc from the Sun.

BVRI CCD photometry of this cluster is done by Alcaino et al. (1994). They determined  $V_{TO} = 19.60 \pm 0.10$  mag with turn off colours at  $B-V = 0.40$ ,  $V-R = 0.27$  and  $V-I = 0.57$  all with estimated external errors of  $\pm 0.06$  mag. By fitting the isochrones, they estimated the cluster age as 16 Gyr. Recently UBV CCD photometric study of the cluster NGC 1904 is done by Kravtsov et al. (1997). They

determined MS turn off at  $V = 19.70 \pm 0.05$  mag,  $B-V = 0.415 \pm 0.01$  mag. The horizontal branch level at the blue edge of the instability strip has  $V = 16.25 \pm 0.10$  mag. They determined the cluster metallicity  $[Fe/H] = -1.76 \pm 0.20$  and age 15 – 19 Gyr.

The cluster parameters derived in earlier studies are given Table 2.

**Table 2.** The cluster NGC 1904 parameters determined in previous photometric studies as follows:

| $(m-M)_V$  | $E(B-V)$  | $V_{TO}$   | $(B-V)_{TO}$ | $V_{HB}$   | Age (Gyr) | References                |
|------------|-----------|------------|--------------|------------|-----------|---------------------------|
| 15.55      | 0.09      |            |              |            |           | Alcaino et al. (1976)     |
|            | 0.01±0.01 |            |              | 16.2±0.1   |           | Stetson & Harris (1977)   |
| 15.60      | 0.02      | 19.4       | 0.52         | 16.2       | 12–18     | Harris et al. (1983)      |
| 15.65      | 0.01      |            |              |            | 16        | Heasley et al. (1986)     |
|            |           | 19.6±0.1   | 0.49         |            | 15–18     | Gratton & Ortolani (1986) |
| 15.45±0.02 | 0.01      | 19.6±0.2   |              | 16.15±0.10 |           | Ferraro et al. (1992)     |
|            |           | 19.6±0.1   | 0.40±0.06    |            | 16        | Alcaino et al. (1994)     |
|            |           | 19.70±0.05 | 0.415±0.01   | 16.25±0.10 | 15–19     | Kravtsov et al. (1977)    |
| 15.63      | 0.01      |            |              | 16.27±0.07 |           | Ferraro et al. (1999)     |

## 1.2 NGC 6341

Being one of the most metal poor globular clusters in the Galaxy ( $[Fe/H] = -2.24$ ; Djorgovski 1993), it provides very good opportunity for studying old and extremely metal poor stars.

Previous photometric studies are those by Arp, Baum & Sandage (1952, 1953), Sandage & Walker (1966), Sandage (1970), Sandage & Katem (1983), Sandage (1983). CCD based photometric studies of this cluster is done by Heasley & Christian (1986) and Stetson & Harris (1988). The cluster parameters derived by them are given in Table 3. VI CCD photometric study has been done recently by Johnson & Bolte (1998) for this cluster. They reported the values for turn-off magnitude  $V_{TO} = 18.80$ , colour of turn-off point  $(V-I)_{TO} = 0.56$  and the magnitude of blue edge of instability strip  $V_{HB} = 15.20$ .

The reddening to the cluster determined by various authors agrees very well with each other. However, the distance and age estimates differ significantly.

GGCs luminosity function has been normally presented by a Gaussian distribution.

The mean absolute magnitude  $M_V \sim -7.3$  magnitude and dispersion  $\sigma(M_V) \sim 1.1$  magnitude (Harris and Racine 1979; Hanes and Whittaker 1987; Racine and Harris 1992; Secker and Harris 1993). However, van den Bergh (1985) has reported that the observed luminosity function of GGCs is slightly asymmetric with a long tail extending upto the faint magnitudes. Recently, Secker (1992) and Racine and Harris (1992) have suggested that GGCs luminosity function is non-Gaussian and attempted to model it as a t-distribution. GGCs luminosity function expressed in terms of clusters per unit luminosity as a truncated power-law model. It is derived by assuming a mass function constructed from three power laws (McLaughlin 1994; Harris and Pudritz 1994).

**Table 3.** The parameters of NGC 6341 determined earlier.

| $(m-M)_V$ | $E(B-V)$         | $V_{TO}$ | $(B-V)_{TO}$ | References                 |
|-----------|------------------|----------|--------------|----------------------------|
| 14.62     | 0.00             |          |              | Sandage & Walker (1966)    |
| 14.63     | 0.02             | 18.43    |              | Sandage (1970)             |
| 14.42     | $0.025 \pm 0.01$ |          |              | Sandage & Katem (1983)     |
| 14.42     | 0.02             |          |              | Sandage (1983)             |
| 14.60     | 0.02             |          |              | Heasley & Christian (1986) |
| 14.60     | 0.02             | 18.77    | 0.394        | Stetson & Harris (1988)    |

This paper is structured as follows: section 2 describes photometric observations and data reductions, section 3 about the colour magnitude diagrams of GGCs NGC 1904 and NGC 6341, section 4 gives the detail information about our artificial add star experiment, section 5 reports luminosity function of GGC NGC 6341, section 6 presents the ages of these GGCs, section 7 describes the relation between GGCs metalicity and their structural parameters and section 8 reports the conclusions of the present work.

## 2 OBSERVATIONS AND DATA REDUCTIONS

The observations of NGC 1904 were carried out in B, V Johnson and R, I Cousins photometric passbands using an TK 1024AB2 CCD detector at the f/3.23 prime focus of 2.34 meter Vainu Bappu Telescope at Vainu Bappu Observatory, Kavalur, India. Each pixel of  $1024 \times 1024$  size CCD is a square of  $24 \mu\text{m}$  size and the entire chip covers a field of  $\sim 11 \times 11 \text{ arcmin}^2$  on the sky. Bias frames are taken at a regular interval. Twilight sky flats were taken for correcting the variation in pixel to pixel response. The read out noise for the CCD system was  $\sim 16$  electrons  $\text{pixel}^{-1}$ , while electrons per ADU was  $\sim 4$ . The details of cluster observations are given in Table 4. The observations of NGC 6341 were carried out in B, V Johnson and I Cousins photometric pass bands using RCA SID 501 thinned back illuminated CCD detector at f/3.29 prime focus of the 2.3 meter Issac Newton Telescope (INT) at La Palma, Canary Islands, Spain. The cluster region was imaged on the nights of 1988 July 21, 22 and 23. In order to avoid saturation of the CCD due to bright stars, we offset the imaged cluster region. It is  $\sim 4.5 \text{ arcmin}$  south from the cluster center. Bias, dark, flat fields, photoelectric standards and other programme clusters regions were also taken on these nights. Nights were good photometric quality. During the observing run of NGC 6341, seeing was  $\sim 1.1$  to  $1.2 \text{ arcsec}$ . At the prime focus, a pixel of  $320 \times 512$  size CCD corresponds to  $0.74 \text{ arcsec}$  square and the entire chip covers a field of view  $\sim 4.0 \times 6.3 \text{ arcmin}^2$  on the sky. The readout noise for the system  $\sim 60$  electrons  $\text{pixel}^{-1}$ , while electrons per ADU was  $\sim 4$ .

Flat field exposures ranging from 1 to 10 seconds in each filter were made of twilight sky. Nine Landolt (1983) photoelectric standards covering a range in brightness ( $10 < V < 12.75$ ) as well as in colour ( $-0.19 < (V - I) < 1.41$ ) were observed for calibration purpose. The detail of cluster observation is given in table 4. Further details of the instrument and observing procedures have been given in Sagar & Griffiths (1991, 1998a, 1998b) and Gupta et al. (2000).

Initial processing of the data frames of NGC 1904 and NGC 6341 were done in the usual manner using standard routines in IRAF package. The evenness of the flat field

frames (summed for each colour band) is better than a few percent in all the filters. The magnitude estimation on each frame has been done using DAOPHOT II and ALLSTAR II profile fitting software (Stetson 1987, 1992). The stellar point spread function (PSF) was evaluated using Penny model of DAOPHOT II from several uncontaminated stars present in each frame. The image parameters and errors provided by DAOPHOT II and ALLSTAR II were used to reject poor measurements. About 10 % stars were rejected in this process. After all the frames were reduced, Stetson's (1992) DAOMATCH and DAOMASTER routines were used for cross identifying the stars measured on different frames of the same cluster region.

**Table 4.** Log of Observations of NGC 1904 and NGC 6341

| Cluster  | Date         | Filter | No. of Exposures $\times$ Exposure Time (in sec.)                   |
|----------|--------------|--------|---|
| NGC 1904 | 1996 Jan. 13 | B      | $1 \times 120, 1 \times 300, 2 \times 1200$                         |
|          | 1996 Jan. 13 | V      | $1 \times 60, 1 \times 180, 2 \times 900$                           |
|          | 1996 Jan. 13 | R      | $1 \times 30, 1 \times 60, 2 \times 600$                            |
|          | 1996 Jan. 13 | I      | $1 \times 30, 1 \times 60, 1 \times 420, 1 \times 600$              |
| NGC 6341 | 1988 July 21 | B      | $1 \times 20, 3 \times 30, 2 \times 110, 1 \times 220$              |
|          | 1988 July 21 | V      | $1 \times 10, 4 \times 20, 1 \times 40, 2 \times 100, 1 \times 200$ |
|          | 1988 July 21 | I      | $4 \times 15, 3 \times 150, 1 \times 250$                           |
|          | 1988 July 22 | B      | $2 \times 20, 4 \times 120$   |
|          | 1988 July 22 | V      | $2 \times 20, 4 \times 100$   |
|          | 1988 July 22 | I      | $2 \times 15, 4 \times 150$   |
|          | 1988 July 23 | B      | $1 \times 20, 1 \times 220$   |
|          | 1988 July 23 | V      | $1 \times 20, 1 \times 220$   |
|          | 1988 July 23 | I      | $1 \times 15, 1 \times 150$   |

The CCD instrumental magnitude of stars in NGC 1904 have been calibrated using local standards in NGC 1904 field (Alcaino et al. 1987). The colour equations for this GGC are given below. In the following equations b, v, r, i represent instrumental magnitudes while B, V, R, I represent standard magnitudes in B, V, R, I passbands

respectively.

$$B = b + (-0.2745 \pm 0.019)(b - v) + (3.236 \pm 0.028) \quad (1)$$

$$V = v + (-0.0102 \pm 0.006)(v - i) + (2.794 \pm 0.008) \quad (2)$$

$$R = r + (-0.0406 \pm 0.038)(v - r) + (2.622 \pm 0.024) \quad (3)$$

$$I = i + (-0.0531 \pm 0.040)(v - i) + (2.677 \pm 0.051) \quad (4)$$

The CCD instrumental magnitudes of stars in NGC 6341 have been calibrated using the colour equations given by Sagar & Griffiths (1991), as the present observations were taken with same equipment during the same period. During observations, the values of atmospheric extinction coefficients in the V passband is determined by Carlsberg Automatic meridian circle between 0.10 and 0.11 mag per unit airmass with almost negligible ( $\sim -0.003$  mag) hourly rate of change of extinction. These along with mean (B–V) atmospheric extinction coefficients for the site were used in determining the colour equations for the CCD system using Landolt (1983) standards (Sagar & Griffiths 1991). Zero points for the B, V and I pass bands cluster frames were determined with respect to nine photoelectric standards of Landolt (1983) by taking into the account the differences in exposure time, atmospheric extinction coefficients and difference between aperture and PSF magnitudes. The zero points are uncertain by 0.02 magnitude in B, V and I pass bands. Errors in the present photometric observations of NGC 1904 and NGC 6341 as a function of brightness are plotted in figure 1.

## 3 COLOUR MAGNITUDE DIAGRAMS

### 3.1 NGC 1904

#### 3.1.1 Overall Morphology of NGC 1904

The V, (B–V); V, (V–R) and V, (V–I) apparent CMDs of 4402 stars observed by us with DAOPHOT II photometric errors in V, (B–V), (V–R) and (V–I) less than 0.07 mag are presented in figure 2. Our CMDs gives the value of turn off mag  $V_{TO} =$



$19.45 \pm 0.05$  mag and turn off colours  $(B-V)_{TO} = 0.39 \pm 0.01$  mag,  $(V-R)_{TO} = 0.27 \pm 0.01$  mag,  $(V-I)_{TO} = 0.58 \pm 0.01$  mag. The turn off level is slightly brighter than the value of  $V_{TO} = 19.6$  mag given by Alcaïno et al. (1994) and  $V_{TO} = 19.7$  mag given by Kravtsov et al. (1997). But it is slightly fainter than the value of  $V_{TO} = 19.4$  mag in Harris et al. (1983).

The present CMDs give the value for HB magnitude  $V_{HB} = 16.00 \pm 0.10$  mag. It is  $0.^m15$  and  $0.^m25$  brighter than the values given by Ferraro et al. (1992) and Kravtsov et al. (1997) respectively. Thus we find  $\Delta V_{TO-HB} = 3.45 \pm 0.11$  mag, close to the average value for Milky Way globular clusters (Buonanno et al. 1989). The blue HB tail in NGC 1904 is one of the longest tail detected in GGCs. CMDs show that there is a HB bump never detected in an intermediate metal poor GGC.

A well defined sub giant branch, red giant branch, asymptotic giant branch, horizontal branch, blue straggler sequence and main sequence are clearly visible in the apparent CMDs (figure 2).

The giant branch is well defined in the CMDs. The tip of branch is located at  $V = 13.28$  mag with colour  $(B-V) = 1.48$  mag,  $(V-R) = 0.77$  mag,  $(V-I) = 1.50$  mag which is about 2.7 mag above the HB level.

The asymptotic giant branch is clearly visible in CMDs and it can be easily separated from red giant branch upto the horizontal branch magnitude level. A well defined sequence of blue straggler is seen above the main sequence turn off in all colours in CMDs (Figure 2).

### 3.1.2 The Red Giant Branch and Metal Abundance

The metallicity of a cluster can be estimated from CMD using the parameters  $\Delta V$  (Sandage and Wallerstein 1960),  $S$  (Hartwick 1968) and  $(B-V)_{0,g}$  (Sandage and Smith 1966). To determine  $\Delta V$ ,  $S$  and  $(B-V)_{0,g}$  cluster's interstellar reddening and magnitude of zero age horizontal branch (ZAHB),  $V_{HB}$  should be known. The interstellar reddening of this cluster is almost negligible with  $E(B-V) \sim 0.01$  mag. We have not calculated independent interstellar reddening of this cluster. Based on its earlier estimates, we adopted the reddening value as  $E(B-V) = 0.01$  for the cluster.

**Table 5.** Metalicity determination from RGB parameters: relations and derived values for NGC 1904.

| Relation                        | Derived Metalicity | Reference                 |
|---------------------------------|--------------------|---------------------------|
|                                 |                    | $(B-V)_{0,g}$             |
| $4.30(B-V)_{0,g} - 5.00$        | $-1.58$            | Zinn and West 1984        |
| $3.84(B-V)_{0,g} - 4.63$        | $-1.57$            | Gratton 1987              |
| $4.68(B-V)_{0,g} - 5.19$        | $-1.46$            | Costar and Smith 1988     |
| $2.85(B-V)_{0,g} - 3.76$        | $-1.49$            | Gratton and Ortolani 1989 |
|                                 |                    | $\Delta V_{1.4}$          |
| $-0.924 \Delta V_{1.4} + 0.913$ | $-1.44$            | Zinn and West 1984        |
| $-1.01 \Delta V_{1.4} + 1.30$   | $-1.28$            | Costar and Smith 1988     |
| $-0.65 \Delta V_{1.4} + 0.28$   | $-1.38$            | Gratton and Ortolani 1989 |
|                                 |                    | S                         |
| $-0.29 S - 0.01$                | $-1.78$            | Gratton and Ortolani 1989 |

Adopting  $E(B-V) = 0.01$ ,  $V_{HB} = 16.00 \pm 0.10$  and the data in Table 5, we derived the reddened colour of RGB at HB level  $(B-V)_{0,g} = 0.80$  mag; difference between the magnitude of the RGB at a de reddened colour of  $(B-V)_0 = 1.4$  mag and the HB  $\Delta V_{1.4} = 2.55$  mag and slope of the RGB  $S = 6.09$ . Using these parameters we derived the values of  $[Fe/H]$  (table 5) using different relations by different authors. The mean value of  $[Fe/H]$  obtained in this way are:

$[Fe/H]_{(B-V)_{0,g}} = -1.52$ ,  $[Fe/H]_{\Delta V_{1.4}} = -1.37$  and  $[Fe/H]_S = -1.78$ . In conclusion, our metalicity estimation covers a range of  $-1.78$  to  $-1.37$  while previous metalicity estimates shows (see Table 2 of Alcaino et al. 1994) the metalicity range for this cluster is  $-1.78$  to  $-1.42$ . Thus we confirm the previous metalicity determinations.

### 3.1.3 Distance to the NGC 1904

HB of a globular cluster is a standard candle for determine the distance modulus. We used the relation between absolute magnitude of HB and metalicity given in the

catalogue of Harris (1994) for determining the  $M_V(\text{HB})$  for the cluster

$$M(\text{HB}) = 0.2[\text{Fe}/H] + 1.0 \quad (5)$$

We have used three different methods ( $(B-V)_{0,g}$ ,  $\Delta V_{1.4}$  and S) for calculating the metallicity of the cluster. The mean value of metallicity determined from the above three methods are  $-1.52$ ,  $-1.37$  and  $-1.78$  respectively. Using the above equation we get the following values of  $M(\text{HB})$  for different values of metallicity:

$$[\text{Fe}/H] = -1.52, M(\text{HB}) = 0.70, (m - M) = 15.30 \quad (6)$$

$$[\text{Fe}/H] = -1.37, M(\text{HB}) = 0.73, (m - M) = 15.27 \quad (7)$$

$$[\text{Fe}/H] = -1.78, M(\text{HB}) = 0.64, (m - M) = 15.36 \quad (8)$$

The isochrone fitting gives the value of  $(m-M) = 15.43$  (see in section Ages of GGCs). Our value for  $(m-M)$  is slightly less than the value reported by Ferraro et al. (1992) which is 15.45. They shifted the values of their fiducial sequences and overlap with M13. Alcaïno et al. (1994) have derived the value of  $(m-M) = 15.75 \pm 0.1$  mag while Stetson and Harris (1977) have reported the value of  $(m-M)$  as 15.60. Both values are slightly larger than the value obtained here.

### 3.1.4 Asymptotic Giant Branch

The AGB is the least studied of all sequences in GGCs CMDs. Generally this branch is not clearly separated in most CMDs of GGCs. Reliable separation between RGB and AGB require good accuracy of data points in CMDs. In the present study AGB is clearly separated from RGB in all CMDs (figure 2.) for  $V$  in range from 13.2 to 15.6 mag.

### 3.1.5 Blue Stragglers

Blue Straggler Stars (BSSs) are found in all populations: in the field, in open clusters of all ages (Population I, young disk, old disk), in globular clusters (Population II, halo) and dwarf galaxies. These stars lie above the main sequence turn off in CMDs, a region where, if BSSs had been normal single stars, they should already have evolved

away from the main sequence. These enigmatic stars lie blueward of turn off and appear to linger or straggler in their evolutionary process, hence the name blue stragglers. The actual positions of BSSs in CMD may have a dependency on metallicity. (Fusi Pecci et al. 1992).

**Table 6.** Fiducial sequences for V vs (B–V), (V–R) and (V–I) colour magnitude diagrams of NGC 1904 displayed in Figure 2. Suffix 1, 2, 3 and 4 represent MS, SGB & RGB, AGB and HB respectively.

| V <sub>1</sub> | (B–V) <sub>1</sub> | V <sub>2</sub> | (B–V) <sub>2</sub> | V <sub>1</sub> | (V–R) <sub>1</sub> | V <sub>2</sub> | (V–R) <sub>2</sub> | V <sub>1</sub> | (V–I) <sub>1</sub> | V <sub>2</sub> | (V–I) <sub>2</sub> |
|----------------|--------------------|----------------|--------------------|----------------|--------------------|----------------|--------------------|----------------|--------------------|----------------|--------------------|
| 19.50          | 0.40               | 18.22          | 0.64               | 19.46          | 0.28               | 18.89          | 0.35               | 19.49          | 0.58               | 18.80          | 0.76               |
| 19.70          | 0.40               | 18.46          | 0.62               | 19.65          | 0.28               | 18.96          | 0.32               | 19.69          | 0.58               | 18.88          | 0.72               |
| 19.86          | 0.41               | 18.68          | 0.60               | 19.80          | 0.28               | 19.06          | 0.30               | 19.84          | 0.59               | 18.93          | 0.68               |
| 20.11          | 0.43               | 18.77          | 0.58               | 20.10          | 0.30               | 19.15          | 0.29               | 20.04          | 0.61               | 18.98          | 0.65               |
| 20.29          | 0.45               | 18.86          | 0.55               | 20.48          | 0.31               | 19.27          | 0.28               | 20.25          | 0.63               | 19.08          | 0.61               |
| 20.54          | 0.47               | 18.94          | 0.50               | 20.61          | 0.32               | 19.38          | 0.28               | 20.50          | 0.65               | 19.18          | 0.60               |
| 20.73          | 0.50               | 19.01          | 0.45               | 20.84          | 0.33               | V <sub>3</sub> | (V–R) <sub>3</sub> | 20.70          | 0.67               | 19.31          | 0.58               |
| 20.93          | 0.53               | 19.18          | 0.41               | 21.03          | 0.35               | 13.25          | 0.75               | 20.83          | 0.68               | V <sub>3</sub> | (V–I) <sub>3</sub> |
| 21.10          | 0.56               | 19.34          | 0.40               | V <sub>2</sub> | (V–R) <sub>2</sub> | 13.46          | 0.72               | 21.00          | 0.71               | 13.26          | 1.43               |
| V <sub>2</sub> | (B–V) <sub>2</sub> | V <sub>3</sub> | (B–V) <sub>3</sub> | 13.33          | 0.76               | 13.67          | 0.68               | V <sub>2</sub> | (V–I) <sub>2</sub> | 13.46          | 1.37               |
| 13.28          | 1.46               | 13.25          | 1.45               | 13.46          | 0.73               | 13.90          | 0.64               | 13.24          | 1.50               | 13.69          | 1.31               |
| 13.45          | 1.38               | 13.41          | 1.37               | 13.69          | 0.69               | 14.05          | 0.62               | 13.36          | 1.45               | 13.97          | 1.23               |
| 13.62          | 1.32               | 13.55          | 1.30               | 13.86          | 0.67               | 14.32          | 0.59               | 13.52          | 1.37               | 14.20          | 1.17               |
| 13.74          | 1.27               | 13.74          | 1.24               | 14.07          | 0.64               | 14.66          | 0.54               | 13.72          | 1.32               | 14.41          | 1.12               |
| 13.95          | 1.20               | 13.97          | 1.16               | 14.32          | 0.61               | 14.85          | 0.52               | 13.97          | 1.26               | 14.71          | 1.08               |
| 14.16          | 1.14               | 14.17          | 1.08               | 14.66          | 0.57               | 15.11          | 0.49               | 14.25          | 1.20               | 14.89          | 1.03               |
| 14.35          | 1.09               | 14.48          | 1.00               | 14.96          | 0.54               | 15.36          | 0.47               | 14.53          | 1.15               | 15.12          | 0.98               |
| 14.54          | 1.05               | 14.72          | 0.87               | 15.21          | 0.53               | 15.61          | 0.44               | 14.74          | 1.11               | 15.37          | 0.94               |
| 14.75          | 1.00               | 15.37          | 0.80               | 15.45          | 0.51               | 15.70          | 0.42               | 15.09          | 1.07               | V <sub>4</sub> | (V–I) <sub>4</sub> |
| 14.97          | 0.96               | 15.61          | 0.70               | 15.68          | 0.50               | V <sub>4</sub> | (V–R) <sub>4</sub> | 15.32          | 1.04               | 15.98          | 0.35               |
| 15.18          | 0.92               | V <sub>4</sub> | (B–V) <sub>4</sub> | 15.85          | 0.49               | 16.04          | 0.11               | 15.60          | 1.00               | 16.06          | 0.29               |
| 15.35          | 0.89               | 16.12          | 0.14               | 16.04          | 0.48               | 16.18          | 0.08               | 15.85          | 0.98               | 16.18          | 0.22               |
| 15.58          | 0.85               | 16.26          | 0.11               | 16.27          | 0.46               | 16.36          | 0.05               | 16.08          | 0.96               | 16.31          | 0.16               |
| 15.94          | 0.81               | 16.50          | 0.06               | 16.50          | 0.45               | 16.61          | 0.01               | 16.34          | 0.93               | 16.49          | 0.11               |
| 16.16          | 0.78               | 16.79          | –0.04              | 16.74          | 0.44               | 16.88          | –0.02              | 16.59          | 0.91               | 16.74          | 0.06               |
| 16.35          | 0.76               | 17.12          | –0.12              | 16.97          | 0.43               | 17.03          | –0.03              | 16.87          | 0.89               | 17.07          | 0.01               |
| 16.61          | 0.74               | 17.48          | –0.16              | 17.22          | 0.42               | 17.35          | –0.05              | 17.15          | 0.87               | 17.35          | –0.01              |
| 16.87          | 0.72               | 17.88          | –0.18              | 17.50          | 0.42               | 17.67          | –0.06              | 17.38          | 0.86               | 17.71          | –0.05              |
| 17.03          | 0.71               | 19.06          | –0.19              | 17.73          | 0.41               | 18.09          | –0.06              | 17.58          | 0.85               | 18.09          | –0.08              |
| 17.27          | 0.69               |                |                    | 17.98          | 0.40               | 18.26          | –0.06              | 17.86          | 0.84               |                |                    |
| 17.56          | 0.68               |                |                    | 18.24          | 0.40               | 18.58          | –0.07              | 18.19          | 0.82               |                |                    |
| 17.79          | 0.66               |                |                    | 18.51          | 0.39               | 18.79          | –0.07              | 18.39          | 0.81               |                |                    |
| 17.99          | 0.65               |                |                    | 18.72          | 0.37               | 19.08          | –0.07              | 18.55          | 0.79               |                |                    |

In the present study we have detected a sample of about 30 BSSs in magnitude range  $17 < V < 18.5$  and colour range  $-0.2 < (B-V) < 0.5$  (in Figure 1). Few BSSs of the present sample are not seen in  $V$  vs  $(V-R)$  and  $V$  vs  $(V-I)$  CMDs due to our error rejection criterion. Since BSSs are more centrally concentrated, the error  $\delta(V-R)$ ,  $\delta(V-I)$  is more than 0.07 for BSSs not detected in these CMDs.

## 3.2 NGC 6341

### 3.2.1 Overall morphology of NGC 6341

The  $V$ ,  $(B-V)$  and  $V$ ,  $(V-I)$  diagrams for all 3570 stars observed by us are presented in Figure 3. The CMDs have well defined turn off (TO) region at  $V_{TO} = 18.6 \pm 0.05$ ,  $(B-V)_{TO} = 0.42$ ,  $(V-I)_{TO} = 0.60$ . The values of HB parameters are  $V_{HB} = 15.17 \pm 0.05$ ,  $(B-V)_{HB} = 0.17$ ,  $(V-I)_{HB} = 0.30$ . Fiducial sequences of  $V$  vs  $(B-V)$  and  $V$  vs  $(V-I)$  CMDs for MS, SGB and RGB are reported in Table 7 and plotted in figure 4 (a) and figure 4 (b) respectively.  $V$  vs  $(B-V)$  and  $V$  vs  $(V-I)$  fiducial points published in the previous studies of the same cluster are also plotted in figure 4 for comparison.

The CMDs of NGC 6341 reveal a relatively steep RGB and a HB population of the blue side of RR Lyrae instability strip. Both of these features are characteristic of a metal poor stellar system.

Previous studies (see table 3) show that the  $E(B-V)$  values for the cluster range from 0.02 to 0.03 mag. So, we adopt the value of  $E(B-V) = 0.02$  mag for the cluster in the present analysis. Keeping the same distance modulus value for  $V$  vs  $(V-I)$  CMD which we reported for  $V$  vs  $(B-V)$  CMD, the best fit of fiducial sequence to theoretical isochrones give the value of  $E(V-I) = 0.05 \pm 0.01$ .

As noted above the HB of NGC 6341 is predominantly blueward of RR Lyrae instability strip. The value of  $V_{HB} = 15.17 \pm 0.05$ . Figure 3 gives the following values of the RGB colour at the level of HB:  $(B-V)_g = 0.72$ . With the adopted reddening of  $E(B-V) = 0.02$  this gives the value of  $(B-V)_{0,g} = 0.70$ . Using the relation given by Gratton (1989)  $[Fe/H] = (2.85 \pm 0.37) (B-V)_{0,g} - (3.76 \pm 0.31)$  which has rms error

of 0.17 dex, we get  $[\text{Fe}/\text{H}] = -1.77 \pm 0.20$ . Previous metallicity determinations of this cluster range from  $-1.77$  to  $-2.24$  (Buonanno et al. 1985).

**Table 7.** Fiducial sequences for V vs (B–V) and (V–I) colour magnitude diagrams of NGC 6341 displayed in Figure 3. Suffix 1, 2 and 3 represent MS, SGB & RGB and HB respectively.

| V <sub>1</sub> | (B–V) <sub>1</sub> | V <sub>2</sub> | (B–V) <sub>2</sub> | V <sub>1</sub> | (V–I) <sub>1</sub> | V <sub>2</sub> | (V–I) <sub>2</sub> |
|----------------|--------------------|----------------|--------------------|----------------|--------------------|----------------|--------------------|
| 18.61          | 0.42               | 16.11          | 0.73               | 18.64          | 0.60               | 18.17          | 0.65               |
| 18.71          | 0.42               | 16.46          | 0.70               | 18.89          | 0.61               | 18.44          | 0.61               |
| 18.97          | 0.44               | 16.70          | 0.69               | 19.02          | 0.62               | 18.46          | 0.60               |
| 19.19          | 0.45               | 16.93          | 0.68               | 19.29          | 0.64               | 18.57          | 0.60               |
| 19.40          | 0.47               | 17.21          | 0.66               | 19.36          | 0.65               | V <sub>3</sub> | (V–I) <sub>3</sub> |
| 19.57          | 0.49               | 17.49          | 0.65               | 19.60          | 0.67               | 15.18          | 0.30               |
| 19.75          | 0.51               | 17.71          | 0.63               | 19.78          | 0.68               | 15.26          | 0.22               |
| 19.92          | 0.53               | 17.90          | 0.61               | 20.05          | 0.72               | 15.45          | 0.14               |
| 20.11          | 0.56               | 18.01          | 0.57               | 20.32          | 0.76               | 15.70          | 0.06               |
| 20.26          | 0.59               | 18.11          | 0.51               | 20.58          | 0.80               | 16.00          | –0.01              |
| 20.37          | 0.61               | 18.20          | 0.48               | 20.85          | 0.86               | 16.29          | –0.06              |
| 20.56          | 0.65               | 18.35          | 0.44               | 21.00          | 0.89               | 16.50          | –0.11              |
| 20.73          | 0.68               | 18.50          | 0.43               | 21.25          | 0.94               |                |                    |
| 20.88          | 0.71               | V <sub>3</sub> | (B–V) <sub>3</sub> | 21.43          | 0.98               |                |                    |
| 21.06          | 0.74               | 15.17          | 0.17               | 21.52          | 1.00               |                |                    |
| 21.21          | 0.78               | 15.26          | 0.11               | 21.74          | 1.06               |                |                    |
| 21.38          | 0.81               | 15.38          | 0.07               | 21.88          | 1.09               |                |                    |
| 21.62          | 0.86               | 15.57          | 0.01               | 22.12          | 1.15               |                |                    |
| 21.81          | 0.90               | 15.85          | –0.02              | V <sub>2</sub> | (V–I) <sub>2</sub> |                |                    |
| 22.00          | 0.93               | 16.08          | –0.06              | 14.20          | 1.09               |                |                    |
| 22.22          | 0.97               | 16.42          | –0.09              | 14.42          | 1.04               |                |                    |
| V <sub>2</sub> | (B–V) <sub>2</sub> |                |                    | 14.76          | 1.01               |                |                    |
| 14.05          | 0.97               |                |                    | 14.87          | 0.99               |                |                    |
| 14.20          | 0.94               |                |                    | 15.16          | 0.96               |                |                    |
| 14.31          | 0.92               |                |                    | 15.43          | 0.94               |                |                    |
| 14.48          | 0.90               |                |                    | 15.74          | 0.91               |                |                    |
| 14.66          | 0.87               |                |                    | 16.19          | 0.88               |                |                    |
| 14.87          | 0.85               |                |                    | 16.41          | 0.87               |                |                    |
| 15.02          | 0.83               |                |                    | 16.79          | 0.85               |                |                    |
| 15.24          | 0.81               |                |                    | 17.06          | 0.84               |                |                    |
| 15.45          | 0.78               |                |                    | 17.37          | 0.82               |                |                    |
| 15.66          | 0.76               |                |                    | 17.75          | 0.79               |                |                    |
| 15.88          | 0.74               |                |                    | 18.04          | 0.72               |                |                    |

### 3.2.2 Distance Modulus

Globular cluster distance can be determined by comparing the magnitude of stars on the MS to a sample of subdwarfs with distance determine via parallax. This technique requires fairly deep photometry to reach sufficiently far down the MS of a given GGC. Since the MS is very well defined in a range of 3 to 3.5 mag fainter than MS turn off, we used best fit of fiducial points to local subdwarfs to determine the distance modulus. We choose nearby subdwarfs with good trigonometric parallaxes.

We used the five subdwarfs with best estimates of absolute magnitudes and colours. Parameters for these stars are reported in Stetson and Harris (1988) and Laird et al. (1988). Data for these stars are given in the Table 8.

The colour of each star was corrected for adopted reddening, leaving the distance modulus  $(m-M)_V$  as free parameter. The fiducial sequence of NGC 6341 was then shifted until a satisfactory fit was obtained. From visual matching of the two sequences, we derived  $(m-M)_V = 14.48 \pm 0.05$ .

**Table 8.** Selected field subdwarfs from those reported in Stetson and Harris (1988) (a) and Laird et al. (1988) (b).

| HD     | $M_{V_a}$ | $(B-V)_a$ | $[Fe/H]_a$ | $M_{V_b}$       | $(B-V)_b$ | $[Fe/H]_b$ |
|--------|-----------|-----------|------------|-----------------|-----------|------------|
| 25329  | 7.16      | 0.86      | -1.33      | $7.17 \pm 0.20$ | 0.865     | -1.33      |
| 103095 | 6.77      | 0.75      | -1.36      | $6.76 \pm 0.09$ | 0.750     | -1.36      |
| 134439 | 7.04      | 0.76      | -1.40      | $6.98 \pm 0.18$ | 0.760     | -1.46      |
| 134440 | 7.41      | 0.87      | -1.52      | $7.36 \pm 0.18$ | 0.850     | -1.54      |
| 201891 | 5.10      | 0.51      | -1.42      | $5.43 \pm 0.32$ | 0.510     | -1.42      |

## 4 ADD STAR EXPERIMENT: A NEW APPROACH

In any discussion regarding the distribution of stars in crowded field photometric regions of the sky as a function of magnitude, it is necessary to first understand the effects of data completeness as a function of magnitude. In general, it is easy to observe the brighter stars compare to the fainter stars. If the observed stellar image

is of a crowded field of the sky then it is necessary to determine how the degree of completeness varies with crowding. Crowded regions of the sky have higher density of stars and several stars are contaminated by other stars.

To determine the true luminosity function from an uncorrected luminosity function it is necessary to perform artificial stars tests to determine the completeness correction and remove the field stars contamination. For GGC NGC 6341 data we have done artificial stars tests and field stars removal and written below in detail in this section of the paper. For artificial stars tests we have suggested a new method and perform the test on GGC NGC 6341 data in the present paper.

How to determine more accurate completeness factor? It is still a question of debate. For this, different authors have suggested different methodologies. A very simple method for this experiment is suggested by Mateo & Hodge (1987) and Mateo (1988). A brief description of the method is described here. Randomly selected artificial stars with a known magnitude range and random or known positions in the original image frames were added. Artificial stars can be generated in the observed image frames by using empirically derived point spread function (PSF) from each image frame using ADDSTAR routine in DAOPHOT II software. Then the image frames with artificially added stars are subjected to identical data reduction process using same PSF as the original frames were done.

Data completeness factor CF is defined by

$$CF = \frac{N_{recovered}}{N_{added}} \quad (9)$$

Authors who have used the method suggested by Mateo & Hodge (1987) and Mateo (1988) believe that the procedure of the added artificial stars should not change the crowding characteristics of the original data frame by limited number ( $\sim 5-15$  % of the number of originally detected stars) can be added at one time. Thus, to have a satisfactory number statistics for the determination of CF, one must has to repeat the above process on a given data frame for many time. It requires a huge amount of the CPU time.

We believe if the observed open cluster is very crowded or if it is a galactic globular



cluster then adding  $\sim 5\text{--}15\%$  stars of the originally detected stars will change the crowding effect and hence, the PSF of the original image frame and the image frame with added stars will vary significantly if we redetermine the PSF of artificially added stars image frame. Since artificial stars were generated by using original image PSF, people use the PSF of original image frames on stars added image frame for recovering the added artificial stars.

Since our final photometry file of the cluster take data from all images frames (short, medium and long exposures at different air masses). We generated artificial stars on all image frames of all passbands in the following way:

1. For generating artificial stars we used ADDSTAR routine of DAOPHOT II software.
2. For individual image frame we check stars detection magnitude limit and then decide the magnitudes of artificial stars.
3. We select the magnitude bin for determining the completeness factor (as example 0.25 magnitude bin). In a box of  $40\text{ pixel} \times 40\text{ pixel}$  we generate one star on every image frame. On one image frame of CCD chip size of  $320\text{ pixel} \times 512\text{ pixel}$ , total 96 artificial stars are generated. This type of five frames are generated for one magnitude bin at one original image frame. The magnitude of these stars on the five frames will vary by 0.05 magnitude which can be used as the central magnitude of magnitude bin limit. Using this method we can give equal weight to all artificially added stars which will show real crowding effect.  $40\text{ pixel} \times 40\text{ pixel}$  box will not overlap with two neighbor artificial stars. In the final photometry file of NGC 6341 we have 3570 stars detected. So, in one image frame 96 artificial stars will not affect its crowding.
4. After generating artificial stars of different magnitude on a specific image frame. We do the photometry of these new frames using the PSF of the original image frame in the same manner as the original image frames. This process will be applied to all original image frames.
5. After doing the photometry on all new frames we determine the ratio of total number of recovered stars and artificially generated stars from all image frames for a specific magnitude bin and a specific passband. It gives the completeness factor

for the central magnitude of the specific magnitude bin and specific passband. This process was applied for all passbands, all magnitude bins and all image frames.

The completeness factor determined in this way for the main sequence stars of the colour magnitude diagram of the GGC NGC 6341 is given in Table 9.

**Table 9.** Completeness factor for the cluster NGC 6341 are presented. CF(B), CF(V) and CF(I) denote the completeness factor on image frames B, V and I respectively. Magnitude denote the calibrated magnitude for B, V and I pass bands.

| Magnitude | CF(B) | CF(V) | CF(I) |
|-----------|-------|-------|-------|
| 18.00     | 0.96  | 0.91  | 0.57  |
| 18.25     | 0.94  | 0.91  | 0.56  |
| 18.50     | 0.92  | 0.89  | 0.52  |
| 18.75     | 0.92  | 0.87  | 0.55  |
| 19.00     | 0.89  | 0.82  | 0.52  |
| 19.25     | 0.85  | 0.75  | 0.45  |
| 19.50     | 0.79  | 0.64  | 0.41  |
| 19.75     | 0.75  | 0.42  | 0.40  |
| 20.00     | 0.65  | 0.26  | 0.35  |
| 20.25     | 0.55  | 0.18  | 0.23  |
| 20.50     | 0.40  |       | 0.16  |
| 20.75     | 0.26  |       |       |
| 21.00     | 0.14  |       |       |

## 5 LUMINOSITY FUNCTION OF NGC 6341

The luminosity function of a GGC can be used to study the cluster’s state of dynamical evolution and initial mass function; together these two factor lead to the cluster’s present day mass function. Since in our data we don’t have sufficient mass range in masses of main sequence stars, so, we were not able to determine the present day mass function of the cluster. From the main sequence of the colour magnitude diagram, luminosity function of NGC 6341 were determined from star counts in the

bin width of 0.25 mag in V passband. The first step towards the estimation of main sequence luminosity function is the estimation of the field star contamination. The contamination by the field stars in the direction of NGC 6341 has been estimated by Ratnatunga & Bahcall (1985) using the Galaxy model given by Bahcall & Soneira (1980). The expected number of field stars in different magnitude bins with different colours are given in table 10.

**Table 10.** Field star contamination correction in the observed galactic globular cluster NGC 6341 field according to model of Ratnatunga & Bahcall (1985).

|                   | V < 15 | 15 < V < 17 | 17 < V < 19 | 19 < V < 21 |
|-------------------|--------|-------------|-------------|-------------|
| B – V < 0.8       | 1      | 2           | 4           | 7           |
| 0.8 < B – V < 1.3 | 0      | 2           | 4           | 3           |
| B – V > 1.3       | 0      | 0           | 3           | 10          |

The main factor which limits the precise determination of luminosity function from observation is data completeness. Data completeness can be estimated by artificial add star experiment which is done in the previous section of the paper by our new method.

The main sequence luminosity function is derived for a bin width of 0.25 mag in V using stars in V vs (B–V) colour magnitude diagram. The procedure used for the data completeness correction has been described in detail earlier by Sagar & Richtler (1991) and again recently by Sagar & Griffiths (1998b). Luminosity function of NGC 6341 is given in the table 11.

Figure 3 shows the luminosity function derived from the original star counts and after the completeness correction and field star subtraction. The slope of the luminosity function is derived as  $0.58 \pm 0.05$  from the histogram plotted in the figure 6.

**Table 11.** Luminosity function of NGC 6341. The counts are taken from the V Vs (B – V) colour magnitude diagram while the completeness factor is taken as minimum of the pair (CF(B), CF(V)). RN denotes star counts in 0.25 magnitude bin width. After the correction of data completeness and field star contamination RN yields N.

| V(mag) | CF   | RN  | log(RN) | N      | log(N) |
|--------|------|-----|---------|--------|--------|
| 18.00  | 0.91 | 21  | 1.32    | 22.08  | 1.34   |
| 18.25  | 0.91 | 32  | 1.51    | 34.16  | 1.53   |
| 18.50  | 0.89 | 41  | 1.61    | 45.07  | 1.65   |
| 18.75  | 0.87 | 66  | 1.82    | 74.36  | 1.87   |
| 19.00  | 0.82 | 59  | 1.77    | 70.95  | 1.85   |
| 19.25  | 0.75 | 76  | 1.88    | 99.33  | 2.00   |
| 19.50  | 0.64 | 94  | 1.97    | 144.87 | 2.16   |
| 19.75  | 0.42 | 131 | 2.12    | 309.90 | 2.49   |

## 6 AGES OF THE GALACTIC GLOBULAR CLUSTERS

The ages of globular clusters has been a topic of great interest for many years. The primary reason for this interest is cosmological: the universe must be older than the objects within it. Milky Way globular clusters contains the oldest stars for which reliable age estimates are available, and thus provide a lower limit of the age of the Universe. Ages of GGCs are debated since long because the lower limit of the age of the universe on the basis of GGCs ages more than the upper limit of the age of the Universe on the basis of some cosmological models.

The well studied GGCs NGC 1904 and NGC 6341 are excellent objects for age determinations. They have high galactic latitudes, so very little interstellar reddening, and low metallicities. The latter is helpful since the stellar models are more reliable at low metallicity.

In order to determine the age of a cluster by theoretical isochrone fitting, one should

know about the main sequence turn off (MSTO) point, distance modulus, redenning of the cluster. These parameters are derived for these GGCs in the previous sections.

## 6.1 NGC 1904

The determination of the distance modulus by fitting the isochrones to fiducial sequences of CMDs of the cluster NGC 1904 gives the value of  $(m-M)_V = 15.43$  mag for redenning values of  $E(B-V) = 0.01$  mag,  $E(V-R) = 0.02$  mag,  $E(V-I) = 0.05$  mag.

In order to derive age of the cluster stars, we converted apparent  $V$ ,  $(B-V)$ ,  $(V-R)$  and  $(V-I)$  diagrams into intrinsic ones. Figure 7 displays plots of the absolute magnitude  $M_V$  against  $(B-V)_0$ ,  $(V-R)_0$  and  $(V-I)_0$ . We have estimated the age of the cluster stars by fitting stellar evolutionary isochrones appropriate for the cluster metallicity given by Demarque et al. (1996) to the fiducial sequences of NGC 1904, we obtained the age of the cluster as  $14 \pm 1$  GYr.

Previous age estimation of this cluster has a ranges from 12 to 19 Gyr (table 2). Our age estimation is about the average of previous age determination of this GGC.

## 6.2 NGC 6341

In order to derive age of the cluster stars, we converted apparent  $V$ ,  $(B-V)$  and  $(V-I)$  diagrams into intrinsic ones. Using the value of distance modulus  $(m-M)_V = 14.48$  and redenning  $E(B-V) = 0.02$  and  $E(V-I) = 0.05$ . Figure 8 displays plots of the absolute magnitude  $M_V$  against  $(B-V)_0$  and  $(V-I)_0$ . We have estimated the age of the cluster by fitting stellar evolutionary isochrones given by Demarque et al. (1996) for the cluster metallicity in figure 8. By fitting the isochrones to the fiducial sequences of NGC 6341, we obtained the age of the cluster is  $16 \pm 1$  GYr.

## 7 RELATION BETWEEN METALICITY AND STRUCTURAL PARAMETERS OF GGCs

Using main sequence luminosity functions of 9 galactic globular clusters (hereafter GGCs), McClure et al. (1986) suggested that the mass function power law index  $x$  of GGCs depends on the metallicity  $[\text{Fe}/\text{H}]$  of the cluster. Pryor et al. (1986) calculated mass segregation corrections for GGCs mass functions using multicomponent King models with power law mass functions to eight observed GGCs mass functions. They found that the corrected mass function exponents still show the previously reported intrinsic variation among GGCs and the correlation of that variation with metallicity. Deep luminosity function studies of the GGC M30 is done by Piotto et al. (1990) using 2.2 meter ESO/MPI telescope and CCD detector. Their data agree well with the trend of  $x$  found by Pryor et al. (1986) and yielded a global mass function  $x_0 \sim 0.7$ .

Capaccioli et al. (1991) took the sample of 14 GGCs with deep CCD luminosity functions and found that formally suggested dependence of  $x$  on  $[\text{Fe}/\text{H}]$  is not supported by the new data. Capaccioli et al. (1991) found a possible trend of the slope with the galactocentric distance  $R$  and with the distance from the galactic disk  $Z_g$  of GGCs. Using original luminosity functions of 17 GGCs data, Capaccioli et al. (1993) derived the mass functions of these GGCs in a consistent way. They interpret the dependence of the mass function slope on the  $R$  and  $Z_g$  as an evidence of a selective loss of stars induced by the GGC dynamical evolution. The same data of a sample of 17 GGCs by Capaccioli et al. (1993) is used by Djorgovski et al. (1993). They analyzed the dependence of stellar mass function slopes for a sample of 17 GGCs on different parameters of GGCs. They found the mass function slopes in the range of  $0.5 \leq (M/M_\odot) \leq 0.8$  are largely determined by  $R$  and  $Z_g$  and also a lesser extent on the GGCs metallicity. Other parameters of GGCs have little effect on the mass function slopes.

Drukier et al. (1988) have done deep luminosity function studies of GGC M13 using CFHT and CCD detector. They found the shape of mass function of this cluster

is inconsistent with the predicted for mass segregation near the position of M13 in the models of Pryor et al. (1986). Mass functions studies of 2 GGCs M13 and M71 down to the main sequence to  $0.2 M_{\odot}$  was done by Richer et al. (1990). They used existing data of GGC NGC 6397 also for covering entire range in metal abundance of GGCs. They reported there is no correlation between the mass function slopes and the cluster metal abundance. Based on observations with the NTT at ESO and the du Pont Telescope at Las Campanas Observatory Richer et al. (1991) determined luminosity and mass function of GGCs  $\omega$  Cen, M5 and NGC 6752. They used their new data of these GGCs and existing data for GGCs M13, NGC 6397 and M71 to investigate any systematics of the cluster mass function slopes and their evolution. They reported a very important and quite robust result is that some (and perhaps all) clusters probably have very steep initial mass function slope exceeding 2.5 (Salpeter value 1.3). No clear dependence of the slope on the GGCs structural parameters or metal abundance is found by these authors.

In the present work we took the data of 64 GGCs from Djorgovski (1993). These GGCs parameters are very accurately known. We plotted log of galactocentric distance of GGCs ( $\log R$  Vs  $[\text{Fe}/\text{H}]$ ) and log of distance from the galactic plane of GGCs ( $\log Z_g$  Vs  $[\text{Fe}/\text{H}]$ ) and found there are clear trend in these two plots. So, we decided to divide all GGCs in five metallicity bins and each bin should contain  $\sim 20\%$  of the GGCs from the sample. We plotted mean and median of  $\log R$  Vs  $[\text{Fe}/\text{H}]$  and  $\log Z_g$  Vs  $[\text{Fe}/\text{H}]$  in the figure 9. Using least square fit we got the following relations:

For Mean Value of  $\log R$ ,  $\log Z_g$  and  $[\text{Fe}/\text{H}]$

$$\log R = -0.344 [\text{Fe}/\text{H}] + 0.297 \quad (10)$$

$$\log Z_g = -0.390 [\text{Fe}/\text{H}] - 0.144 \quad (11)$$

For Median Value of  $\log R$ ,  $\log Z_g$  and  $[\text{Fe}/\text{H}]$

$$\log R = -0.226 [\text{Fe}/\text{H}] + 0.462 \quad (12)$$

$$\log Z_g = -0.353 [\text{Fe}/\text{H}] - 0.105 \quad (13)$$

**Table 12.** Mean and Median of Globular Cluster Parameters

| Mean        |             |              | Limits of              | No. of |
|-------------|-------------|--------------|------------------------|--------|
| log(R)      | log $Z_g$   | [Fe/H]       | [Fe/H]                 | GGCs   |
| 0.525±0.294 | 0.099±0.298 | −0.604±0.176 | [Fe/H] > −1.00         | 13     |
| 0.809±0.368 | 0.453±0.484 | −1.318±0.071 | −1.40 ≤ [Fe/H] < −1.00 | 12     |
| 0.748±0.477 | 0.389±0.513 | −1.520±0.060 | −1.60 ≤ [Fe/H] < −1.40 | 15     |
| 0.779±0.328 | 0.391±0.456 | −1.691±0.067 | −1.80 ≤ [Fe/H] < −1.60 | 13     |
| 1.087±0.468 | 0.733±0.549 | −2.019±0.137 | [Fe/H] < −1.80         | 11     |
| Median      |             |              | Limits of              | No. of |
| log(R)      | log $Z_g$   | [Fe/H]       | [Fe/H]                 | GGCs   |
| 0.600±0.294 | 0.110±0.298 | −0.590±0.176 | [Fe/H] > −1.00         | 13     |
| 0.880±0.368 | 0.525±0.484 | −1.315±0.071 | −1.40 ≤ [Fe/H] < −1.00 | 12     |
| 0.710±0.477 | 0.260±0.513 | −1.540±0.060 | −1.60 ≤ [Fe/H] < −1.40 | 15     |
| 0.710±0.328 | 0.370±0.456 | −1.670±0.067 | −1.80 ≤ [Fe/H] < −1.60 | 13     |
| 1.020±0.468 | 0.730±0.549 | −2.019±0.137 | [Fe/H] < −1.80         | 11     |

## 8 CONCLUSIONS

The main conclusions of the present paper are as follows:

1. Using B,V,R,I photometry of NGC 1904, we derived  $E(V-R) = 0.02$  mag,  $E(V-I) = 0.05$  mag,  $V_{HB} = 16.00 \pm 0.10$  mag,  $(B-V)_g = 0.81$  mag,  $(B-V)_{0,g} = 0.80$  mag. Metalicity  $[Fe/H] = -1.52, -1.37$  and  $-1.78$  were derived by using  $(B-V)_{0,g}$ ,  $\Delta V_{1.4}$  and S methods respectively.  $(m-M)_V = 15.30, 15.27$  and  $15.36$  were derived by using  $(B-V)_{0,g}$ ,  $\Delta V_{1.4}$  and S methods respectively. The values of morphological parameters of the cluster are  $V_{TO} = 19.45 \pm 0.05$  mag,  $(B-V)_{TO} = 0.39 \pm 0.01$  mag,  $(V-R)_{TO} = 0.27 \pm 0.01$  mag,  $(V-I)_{TO} = 0.58 \pm 0.01$  mag and  $\Delta V_{TO-HB} = 3.45 \pm 0.11$  mag.
2. Using B,V,I photometry of NGC 6341, we derived  $E(V-I) = 0.05$ ,  $V_{HB} = 15.17 \pm 0.05$  mag,  $(B-V)_{HB} = 0.17$  mag,  $(V-I)_{HB} = 0.30$  mag,  $(B-V)_g = 0.72$  mag,  $(V-I)_g = 0.86$  mag,  $(B-V)_{0,g} = 0.70$  mag,  $(V-I)_{0,g} = 0.81$  mag, metalicity  $[Fe/H] = -1.77 \pm 0.20$  by using  $(B-V)_{0,g}$ ,  $(m-M)_V = 14.48 \pm 0.05$  mag from subdwarf fitting.



The value of  $V_{TO} = 18.60 \pm 0.05$  mag,  $(B-V)_{TO} = 0.42$  mag and  $(V-I)_{TO} = 0.60$  mag and  $\Delta V_{TO-HB} = 3.43 \pm 0.10$  mag.

**3.** We have suggested a new, add star experiment technique. While its implementation is still in the preliminary stage. We have successfully used it to determine the completeness factors in B, V and I passbands data of GGC NGC 6341. In the near future we will perform the same test on other GGCs data. Using our new method of artificial stars, we added a total of 141120 artificial stars in 1470 separate runs of ADDSTAR routine of DAOPHOT II. For each of the 1470 artificial stars test runs, 96 stars were added to all 49 image frames of B, V and I passbands of the GGC NGC 6341. The above process took about a week long time and 100 % CPU time on a Sun Sparc 1 workstation.

**4.** We determined the main sequence luminosity function of NGC 6341 down to  $V \sim 21.0$  ( $M_V \sim 6.5$ ). The slope of luminosity function is derived as  $0.58 \pm 0.05$ .

**5.** By using theoretical isochrone fitting we determined the ages of GGCs NGC 1904 and NGC 6341 to  $14.0 \pm 1.0$  Gyr and  $16.0 \pm 1.0$  Gyr respectively.

**6.** In the present paper we have tried to solve the controversy about the dependence of mass function power law index of GGCs on metallicity or  $R$  and  $Z_g$ . Figure 9 and equations 10–13 shows that there is clear trend in the plots  $\log R$  Vs.  $[Fe/H]$  and  $\log Z_g$  Vs.  $[Fe/H]$ . We conclude that  $R$ ,  $Z_g$  and  $[Fe/H]$  are not three independent parameters of GGCs but  $R$  and  $Z_g$  depends on  $[Fe/H]$ .

Till date we have mass function study of a few  $\sim 20$  % GGCs only. As sample of mass function measurements of GGCs will increase, we will have better opportunity to obtain more accurate estimation on the empirical correlation between GGCs fundamental parameters and their mass functions slope. At present we can not say confidently its dependent on any fundamental parameter of GGCs.

## ACKNOWLEDGMENTS

I am grateful to Prof. Ram Sagar for providing me his photometric data of NGC 6341 (M92) and valuable suggestions. This work was supported in part by DST Project No. SP/S2/011/93 (PRU) dated 02.01.1997 and Department of Atomic Energy, Government of India.

## FIGURE CAPTIONS

**Figure 1.** Errors in the photometric observations of NGC 1904 and NGC 6341 as a function of brightness.

**Figure 2.**  $V$  vs  $(B - V)$ ,  $V$  vs  $(V - R)$  and  $V$  vs  $(V - I)$  colour magnitude diagrams of the galactic globular cluster NGC 1904.

**Figure 3.**  $V$  vs  $(B - V)$  and  $V$  vs  $(V - I)$  colour magnitude diagrams of the galactic globular cluster NGC 6341.

**Figure 4.** (a) Comparison between the fiducial points of  $V$  vs  $(B - V)$  CMD of NGC 6341 from the present work (solid line), Sandage (1970) (dots), Heasley & Christian (1986) (short dashed), Stetson & Harris (1988) (long dash). (b) Fiducial points of  $V$  vs  $(V - I)$  CMD of the present work (solid line) and by Johnson & Bolte (1998) (dots).

**Figure 5.** Fit of the fiducial sequence of NGC 6341 to the field subdwarfs. Left: data from Stetson & Harris (1988); Right: data from Laird et al. (1988).

**Figure 6.** Luminosity function of main sequence stars in  $V$  Vs  $(B - V)$  colour magnitude diagram of NGC 6341. Dotted line represents the log of actual counts of stars in the colour magnitude diagram. Continuous line represents the log of count after applying completeness correction and field star contamination.

**Figure 7.** Isochrone fitting to the fiducial points of NGC 1904. Fit with the isochrone by Demarque et al. (1996). A good fit is obtained for age 14 Gyr for the CMDs of the present study.

**Figure 8.** Isochrone fitting to the fiducial points of NGC 6341. Fit with the isochrone by Demarque et al. (1996). A good fit is obtained for age 16 yr for the CMDs of the present study.

**Figure 9.** Data of the physical parameters of 64 GGCs from Djorgovski (1993) are plotted. Mean of data is plotted by filled circle and solid line. Median of the data is plotted by open circle and dotted line.

## REFERENCES

- Alcaino, G., 1976, A&AS, 26, 353
- Alcaino, G., Liller, W., Alvarado, F., 1987, AJ, 93, 464
- Alcaino, G., Liller, W., Alvarado, F., Wenderoth, E., 1994, AJ, 107, 230
- Arp, H. C., Baum, W. A., Sandage, A. R., 1952, AJ, 57, 4
- Arp, H. C., Baum, W. A., Sandage, A. R., 1953, AJ, 58, 4
- Bahcall, J. N., Soneria, R. M., 1980, ApJS, 44, 73
- Buonanno, R., Corsi, C. E., Fusi Pecci, F., 1985, A&A, 145, 97
- Buonanno, R., Corsi, C. E., Fusi Pecci, F., 1989, A&A, 216, 80
- Capaccioli, M., Ortolani, S., Piotto, G., 1991, A & A, 244, 298
- Capaccioli, M., Piotto, G., Stiavelli, M., 1993, MNRAS, 261, 819
- Corodoni, J. P., Auriere, M., 1983, A&AS, 54, 431
- Costar, D., Smith, H. A., 1988, AJ, 96, 1925
- Demarque, P., Chaboyer, B., Guenther, D., et al. 1996, private communication
- Djorgovski, S. G. 1993, in Structure and Dynamics of Globular Clusters by S. G. Djorgovski & G. Meylan eds. ASP conf. series 50, p. 373
- Djorgovski, S., Piotto, G., Capaccioli, M., 1993, AJ, 105, 2148
- Drukier, G. A., Fahlman, G. G., Richer, H. B., 1988, AJ, 95, 1415
- Ferraro, F. R., Clementini, G., Fusi Pecci, F., et al. 1992, MNRAS, 256, 391
- Ferraro, F. R., Messino, M., Fusi Pecci, F., et al. 1999, AJ, 118, 1738
- Fusi Pecci, F., Ferraro, F. R., Corsi, C. E., et al. 1992, AJ, 104, 1831
- Goranskij, V. P., 1976, Astron. Tsirk., 902, 5
- Gratton, R. G., 1987, A&A, 179, 181
- Gratton, R. G., Ortolani, S., 1986, A&AS, 65, 63
- Gratton, R. G., Ortolani, S., 1989, A&A, 211, 41
- Gupta, A. C., Subramaniam, A., Sagar, R., Griffiths, W. K., 2000, A & AS, 145, 365
- Hanes, D. A., Whittaker, D. G., 1987, AJ, 94, 906
- Harris, W. E., Pudritz, R. E., 1994, ApJ, 429, 177
- Harris, W. E., Racine, R., 1979, ARAA, 17, 241
- Harris, W. E., Hesser, J. E., Atwood, B., 1983, PASP, 95, 951

Hartwick, F. D. A., 1968, ApJ, 154, 475

Heasley, J. N., Christian, C. A., 1986, ApJ, 307, 738

Heasley, J. N., Janes, K. A., Christian, C. A., 1986, AJ, 91, 1108

Johnson, J. A., Bolte, M., 1998, AJ, 115, 693

Kravtsov, V., Ipatov, A., Samus, N., et al. 1997, A&AS, 125, 1

Laird, J. B., Carney, B. W., Latham, D. W., 1988, AJ, 95, 1843

Landolt, A. U., 1983, AJ, 88, 439

Mateo, M., 1988, ApJ, 331, 261

Mateo, M., Hodge, P., 1987, ApJ, 320, 626

McClure, R. D., Vandenberg, D. A., Smith, G. H., Fahlman, G. G., Richer, H. B.,  
Hesser, J. E., Harris, W. E., Stetson, P. B., Bell, R. A., 1986, ApJL, 307, L49

McLaughlin, D. E., 1994, PASP, 106, 47

Piotto, G., King, I. R., Capaccioli, M., Ortolani, S., Djorgovski, S., 1990, ApJ, 350,  
662

Pryor, C., Smith, G. H., McClure, R. D., 1986, AJ, 92, 1358

Racine, R., Harris, W. E., 1992, AJ, 104, 1068

Ratnatunga, K. U., Bahcall, J. N., 1985, ApJS, 59, 63

Richer, H. B., Fahlman, G. G., Buonanno, R., Fusi Pecci, F., 1990, ApJ, 359, L11

Richer, H. B., Fahlman, G. G., Buonanno, R., Fusi Pecci, F., Searle, L., Thompson,  
I. B., 1991, ApJ, 381, 147

Sagar, R., Richtler, T., 1991, A&A, 250, 324

Sagar, R., Griffiths, W. K., 1991, MNRAS, 250, 683

Sagar, R., Griffiths, W. K., 1998a, MNRAS, 299, 1

Sagar, R., Griffiths, W. K., 1998b, MNRAS, 299, 777

Sandage, A., 1970, ApJ, 162, 841

Sandage, A., 1983, AJ, 88, 1159

Sandage, A., Katem, B., 1983, AJ, 88, 1146

Sandage, A. R., Smith, L. L., 1966, ApJ, 144, 886

Sandage, A., Walker, M. F., 1966, ApJ, 143, 313

Sandage, A. R., Wallerstein, G., 1960, ApJ, 131, 598

Secker, J., 1992, AJ, 104, 1472  
Secker, J., Harris, W. E., 1993, AJ, 105, 1358  
Stetson, P. B., 1987, PASP, 99, 191  
Stetson, P. B., 1992, IAU col. 136 on stellar photometry – current techniques and future developments, eds. C. J. Butler and I. Elliot, p. 291  
Stetson, P. B., Harris, W. E., 1977, AJ, 82, 954  
Stetson, P. B., Harris, W. E., 1988, AJ, 96, 909  
van den Bergh, S., 1985, ApJ, 297, 361  
Zinn, R. J., West, M. J., 1984, ApJS, 55, 45

This figure "gupta\_fig1.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig2.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig3.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>



This figure "gupta\_fig4.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig5.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig6.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig7.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig8.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>

This figure "gupta\_fig9.jpg" is available in "jpg" format from:

<http://arXiv.org/ps/astro-ph/0311443v1>